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Effects of external fields on the Lamb shift

P L KNIGHT

School of Mathematical and Physical Sciences, University of Sussex, Brighton, BN1 9QH, Sussex, UK

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Abstract. It is shown that an external radiation field interacting with an atom alters the value of the Lamb shift. The effect of various spectral profiles is calculated with particular reference to the existence of an electromagnetic mass shift and changes in the Bethe formula. Thermal corrections due to black body radiation are considered and are shown never to be important, in contrast with previous work. Mass renormalization due to external field interactions is shown not to be a useful concept. We conclude that the only important modification is due to the interaction with excitation and decay photons, and that this is likely to be very small in most experiments.

1. Introduction

A bound electron when interacting with an electromagnetic field has its energy altered by its interaction. If the field is generated by the electron's own current the subsequent change in the electron's properties is well understood and is responsible for Lamb shifts and g factor modifications (after renormalization). The same techniques applied to the vacuum corrections will also apply to the coupling with external fields of interest in atomic physics. The contribution of vacuum radiative corrections due to the electromagnetic interaction of a bound state has been calculated to great accuracy and compares very favourably with experiments to date (Brodsky and Drell 1970). There is every prospect of increased experimental accuracy in the not too remote future. So it becomes important to check the contribution of small environmental corrections to the Lamb shift—for example, due to any real radiation fields present. All of such environmental corrections will be at very low energy where the binding is important and will affect the Bethe part of the Lamb shift. Some work has recently appeared in the literature on environmental corrections but suffers from several drawbacks. These are commented on and amended in this work.

In § 2 we set up the basic formalism describing the changes in level shifts due to real photons. In particular we keep both resonant and antiresonant terms in the second order shift, and the quadratic term in first order. This proves crucial in cancelling certain terms which have been identified with electromagnetic mass shifts in the past (Walsh 1971, Bullough and Caudrey 1971). In § 3 the effects of black body radiation are evaluated in both the high and low temperature limits and previous work amended (Walsh 1971, Auluck and Kothari 1952). The result, as we would expect, is too small to be of experimental significance. The effect of a rather artificial spectral profile $\bar{n}_k = |\alpha|^2$, $a < k < b$ is studied in § 4 and the results compared with those of Bullough and Caudrey (1971). Finally we evaluate the effects of Lyman excitation and decay radiation.

In all of these cases we find a cancellation of leading terms such that the atom conspires to interact far less strongly with electromagnetic fields than a free electron. This comes as no surprise: the scattering amplitude changes from the Thomson amplitude $(-\alpha/m)$ for a free electron to the Rayleigh amplitude $\omega^2\Pi(\omega^2)$ for a bound electron ($\Pi(\omega^2)$ is the dynamic polarizability). This reflects a cancellation of the direct term $(-\alpha/m)$ by the binding via the Thomas–Reiche–Kuhn sum rule, and changes the surviving term's frequency dependence. The Lamb shift is caused by the high frequency terms $\hbar ck > (E_i - E_0)$ where such a cancellation is inoperable. The final result for the low energy modification is small for this reason.

2. Changes in level shifts due to real photons

Here we formulate the problem for a simple nonrelativistic atom. We consider the nuclear mass to be infinite, thus neglecting recoil, and ignore all complications due to spin. We calculate the difference between the Lamb shift in the presence of real photons and the Lamb shift due to the vacuum fluctuations only. This could be done directly by subtracting a counter term (Barton 1970); we prefer to consider that part of the Lamb shift in the presence of real photons that is due to the vacuum (spontaneous emission) to be treated in the standard way and isolate the external field dependent part. The interaction Hamiltonian is

$$H_{\text{int}} = -\frac{eA(x)\,.\,p}{m} + \frac{e^2A^2(x)}{2m} \tag{2.1}$$

and the radiation field is expanded into plane waves

$$A(x) = \frac{1}{\sqrt{V}}\sum_{k,\lambda}\left(\frac{2\pi}{|k|}\right)^{1/2}\{a_{k,\lambda}\exp(\mathrm{i}k\,.\,x)\epsilon_k^{(\lambda)} + \mathrm{hc}\} \tag{2.2}$$

where the summation is taken over both polarization $\epsilon_k^{(\lambda)}$ and the wavevector k. We use units $\hbar = c = 1$. Then the vacuum corrections for bound states are (Power 1964)

$$\Delta_1 = \frac{e^2}{2m}\langle m|A^2|m\rangle = \frac{\alpha}{\pi m}\int k\,\mathrm{d}k \tag{2.3}$$

$$\Delta_2 = \frac{e^2}{m^2}\sum_I \frac{\langle m|p\,.\,A|n,k\rangle\langle n,k|p\,.\,A|m\rangle}{E_m-(E_n+k)}$$

$$= \frac{-2}{3\pi}\frac{\alpha}{m^2}\sum_n|\langle m|p|n\rangle|^2\int\left(1-\frac{1}{1+k/(E_n-E_m)}\right)\mathrm{d}k. \tag{2.4}$$

By separating out Δ_2 into binding-dependent and binding-independent terms we get the transverse part of the electron self-mass and the radiative level shift.

With real external fields the above are modified by a photon occupation of mode k. The emission term, represented by (2.4), is now multiplied by $\bar{n}_k + 1$, where $\bar{n}_k$ is the mean number of photons in the mode k, and we now have an additional absorption term, equivalent to (2.4) multiplied by $\bar{n}_k$. The use of $\bar{n}_k$ implies equal populations of photons in both states of polarization, appropriate to thermal radiation. Then the external field dependent Δ_2, after doing the angular integration and assuming that convergence allows the frequency integral to go to infinity is

$$\Delta_2 = \frac{-2\alpha}{3\pi}\sum_n |\boldsymbol{v}_{nm}|^2 \int_0^\infty \mathrm{d}k\, k\bar{n}_k \left(\frac{1}{E_n - E_m + k} + \frac{1}{E_n - E_m - k}\right)$$

where we have included both resonant and antiresonant terms. Combining energy denominators

$$\Delta_2 = \frac{-4\alpha}{3\pi}\sum_n |\boldsymbol{v}_{mn}|^2 \int_0^\infty \mathrm{d}k\, k\bar{n}_k \left(\frac{\Delta E}{(\Delta E)^2 - k^2}\right) \tag{2.5}$$

$$\Delta E \equiv E_n - E_m.$$

Similarly

$$\Delta_1 = \frac{2\alpha}{\pi m}\int k\bar{n}_k\, \mathrm{d}k. \tag{2.6}$$

It is essential that all contributions are included: many papers have been published using only one of these and arriving at misleading results. Indeed the form of (2.5) indicates a departure from a naive generalization of the Bethe result. An alternative is to use the completely equivalent dipole interaction $\mathscr{H} = -e\boldsymbol{r}\,.\,\boldsymbol{E}$, which combines both (2.5) and (2.6) from the beginning.

3. Effects of black body radiation on the Lamb shift

An example of environmental corrections is the thermal contribution when an atom interacts not only with the vacuum field but also the black body radiation present at finite temperatures. This has been considered by Walsh (1971) who takes only Δ_2 and understandably derives a state-independent 'mass term', and by Auluck and Kothari (1952) who take only the first part of Δ_2, deriving the wrong dependence on k and thus on temperature. Inclusion of both the other part of Δ_2 and of Δ_1 cancels their term (odd in T), and Walsh's mass term. The final shift Δ is small since the peak of the thermal photon distribution at 'normal' temperatures is less than the average excitation energy to the continuum which dominate the Lamb shift.

For black body radiation (T in energy units)

$$\bar{n}_k = \left\{\exp\left(\frac{k}{T}\right) - 1\right\}^{-1}$$

$$\Delta_2 = \frac{-4\alpha}{3\pi}\sum_n |\boldsymbol{v}_{nm}|^2 \int_0^\infty k\,\mathrm{d}k \left\{\exp\left(\frac{k}{T}\right) - 1\right\}^{-1} \left(\frac{\Delta E}{(\Delta E)^2 - k^2}\right). \tag{3.1}$$

At low temperatures $k^2 \ll (\Delta E)^2$ for all k^2 since ΔE includes only electric dipole transitions. So

$$\Delta_2 = \frac{-4\alpha}{3\pi}\sum_n \frac{|\boldsymbol{v}_{mn}|^2}{\Delta E} \int_0^\infty k\,\mathrm{d}k \left\{\exp\left(\frac{k}{T}\right) - 1\right\}^{-1} \left\{1 + \left(\frac{k}{\Delta E}\right)^2 \cdots\right\} \tag{3.2}$$

and the first part of Δ_2 is

$$\Delta_2^{(1)} = \frac{-4\alpha}{3\pi}\sum_n \frac{|\boldsymbol{v}_{mn}|^2}{\Delta E}\frac{\pi^2}{6}T^2. \tag{3.3}$$

The sum may be performed using

$$\frac{|v_{mn}|^2}{E_n-E_m} = -(E_m-E_n)|\langle m|r|n\rangle|^2 \tag{3.4}$$

and the dipole sum rule

$$\sum_n |\langle m|r|n\rangle|^2(E_m-E_n) = -\frac{3}{2m} \tag{3.5}$$

giving

$$\Delta_2^{(1)} = -\frac{1}{3}\frac{\pi\alpha T^2}{m} \tag{3.6}$$

that is, equal and opposite in sign to Walsh's T^2 term. This shift is the same for all levels and is equivalent to the black body version of $\delta m_{\rm tr}$. But to assign it to a mass is misleading, since it would imply that spin magnetic moments would change via the Bohr magneton, and a recent experiment shows this is not the case (Mowat 1969). It is best to think of $\Delta_2^{(1)}$ as an energy shift. Here, the distinction is academic since $\Delta_2^{(1)}$ is equal and opposite to Δ_1 and exactly cancels it in the low temperature limit

$$\Delta_1 = \frac{2\alpha}{\pi m}\int_0^\infty k\,\mathrm{d}k\left\{\exp\left(\frac{k}{T}\right)-1\right\}^{-1} = \frac{1}{3}\frac{\pi\alpha T^2}{m}. \tag{3.7}$$

So at low temperatures the Thomson part cancels that part of the $\boldsymbol{p}\,.\,\boldsymbol{A}$ interaction which would have yielded a T^2 term. Such a cancellation also removes the mass shift of Reiss (1966) when applied to a bound state. The only survivor of Δ for a bound electron at low temperatures is $\Delta_2^{(2)}$:

$$\begin{aligned}\Delta_2^{(2)} &= \frac{-4\alpha}{3\pi}\sum_n \frac{|v_{mn}|^2}{(E_n-E_m)^3}\int_0^\infty k^3\,\mathrm{d}k\left\{\exp\left(\frac{k}{T}\right)-1\right\}^{-1} \\ &= \frac{-4\alpha}{3\pi}\sum_n \frac{|v_{mn}|^2}{(E_n-E_m)^3}\frac{T^4\pi^4}{15}.\end{aligned} \tag{3.8}$$

This is Walsh's equation (6) but with the sign reversed. A clearer interpretation can be made if the polarizability is used

$$\Pi_0 = \frac{2\alpha}{3}\sum_n \frac{|\langle n|v|m\rangle|^2}{(E_n-E_m)^3} \tag{3.9}$$

$$\Delta_2^{(2)} = \frac{-2\pi^3}{15}\Pi_0 T^4. \tag{3.10}$$

Now the mean energy density $\bar{u}(T)$ of black body radiation is $-\pi^2T^4/15$ (in our units) and may be related to the mean square electric field of the cavity radiation $\langle \bar{E}^2\rangle$. The term $\Delta_2^{(2)}$ could be loosely described as a dynamic Stark shift due to the radiation (Pancharatnam 1966). For a free electron, Δ_1 is the only surviving shift since the two $\boldsymbol{p}\,.\,\boldsymbol{A}$ terms exactly cancel at low frequencies when one neglects recoil terms.

At high temperatures $T^2 > \Delta E$ we would expect the shift to be identical to the free electron result: at such frequencies the binding is unimportant and the atomic electron scatters as if free. This could be computed with Δ_1 and Δ_2 separately and we might expect a term like $\ln(T)$ as well as a T^2 term. But in view of the interplay of the effects of Δ_1 and Δ_2 we prefer to combine them by using the electric dipole form $-e\boldsymbol{r}.\boldsymbol{E}$:

$$\Delta = \frac{-4\alpha}{3\pi}\sum_n (E_n - E_m)|\boldsymbol{r}_{mn}|^2 T^2 \int \frac{x^3\,\mathrm{d}x}{(\mathrm{e}^x - 1)\{(\Delta E/T)^2 - x^2\}}. \tag{3.11}$$

This produces the low temperature T^4 result (3.10) for $T < \Delta E$. But for the high temperature term $T > \Delta E$ (but still small enough for retardation and relativistic effects to be negligible)

$$\Delta = \frac{-4\alpha}{3\pi}\sum_n (E_n - E_m)|\boldsymbol{r}_{mn}|^2 T^2 \int \frac{x\,\mathrm{d}x}{\mathrm{e}^x - 1}$$

$$= \frac{-4\alpha}{3\pi}\sum_n (E_n - E_m)|\boldsymbol{r}_{mn}|^2 T^2 \frac{\pi^2}{6} = \frac{1}{3}\frac{\alpha\pi T^2}{m} \tag{3.12}$$

which is identical to the free electron result. All the levels are equally shifted and no experimental observation could detect this astronomically at promising stellar temperatures, even if a metastable hydrogenic atom could survive in such an environment.

Barton (1972) has pointed out that there is an extra term not previously considered. To the order in e that we are considering, the $^2\mathrm{S}_{1/2}$ and $^2\mathrm{P}_{1/2}$ states are degenerate. But the external field will mix into the $^2\mathrm{S}_{1/2}$ state the $^2\mathrm{P}_{1/2}$ and $^2\mathrm{P}_{3/2}$ states. These are not contained in the Lamb shift itself which precludes these states, or in the change Δ which does not admit degenerate states of the same n to this order in e. Considering these states separately and allowing them their vacuum splitting (ie formally going to higher order) then this degenerate mixing must be calculated in the high temperature limit since $\Delta E(2\mathrm{S}-2\mathrm{P}) \ll T$ (whilst all other contributions are in the $\Delta E > T$ limit). Therefore

$$\Delta(\text{mixing}) = \frac{4\alpha}{3\pi}(E_n - E_m)|\boldsymbol{r}_{mn}|^2 T^2 \int_0^\infty \frac{x\,\mathrm{d}x}{\mathrm{e}^x - 1}$$

$$= \frac{4\alpha}{3\pi}(E_n - E_m)|\boldsymbol{r}_{mn}|^2 T^2 \frac{\pi^2}{6}. \tag{3.13}$$

For the $^2\mathrm{S}_{1/2} - {}^2\mathrm{P}_{1/2}$ mixing, $E_n - E_m = L$, the Lamb shift. The $^2\mathrm{P}_{1/2}$ and $^2\mathrm{S}_{1/2}$ levels are shifted equally in opposite directions and produce a change in the Lamb shift of 2Δ (mixing). Substituting $|\langle {}^2\mathrm{S}_{1/2}|\boldsymbol{r}|{}^2\mathrm{P}_{1/2}\rangle|^2 = 9a_0^2 = 9/(Z\alpha m)^2$, where a_0 is the Bohr radius

$$\frac{\delta L}{L} = \frac{4\pi T^2}{\alpha Z^2 m^2}. \tag{3.14}$$

But since Δ(mixing) is proportional to $E_n - E_m$, the largest term is the $^2\mathrm{S}_{1/2} - {}^2\mathrm{P}_{3/2}$ mixing:

$$\Delta({}^2\mathrm{S}_{1/2} \to {}^2\mathrm{P}_{3/2}) = \frac{4\alpha}{3\pi}(E({}^2\mathrm{P}_{3/2}) - E({}^2\mathrm{S}_{1/2}))|\boldsymbol{r}_{mn}|^2 T^2 \frac{\pi^2}{6}$$

$$|\langle {}^2\mathrm{S}_{1/2}|\boldsymbol{r}|{}^2\mathrm{P}_{3/2}\rangle|^2 = 18a_0^2 \qquad (E({}^2\mathrm{P}_{3/2}) - E({}^2\mathrm{P}_{1/2})) \simeq \frac{(Z\alpha)^4 m}{32}.$$

Hence

$$\Delta({}^2\mathrm{S}_{1/2} \to {}^2\mathrm{P}_{3/2}) = \frac{\pi\alpha^3 Z^2}{8}\frac{T^2}{m}. \tag{3.15}$$

This is about ten times bigger than the $^2P_{1/2}$ mixing but is still negligible. So we conclude that such thermal shifts, interesting as they are, turn out to be unobservable.

4. Effects of an isotropic field

In this section we consider the effects of an admittedly artificial field distribution $\bar{n}_k = |\alpha_0|^2, a < k < b$. Bullough and Caudrey (1971) have discussed the effects of such a spectral profile on the level shifts of a two-level atom. Their paper is mainly concerned to show how a semiclassical boson assumption for the dipole operators eliminates the Lamb shift, whilst a fermion assumption restores it. Their theory is restricted to a two-level atom and to the decorrelation procedure used. The result is an all-order perturbation theory and their results are not necessarily to be identified with our Δ_1 and Δ_2, although a connection may be drawn as they themselves imply. We shall for the purposes of this section assume such a correspondence.

They find a shift in the two-level splitting ω_{os} of

$$\Delta(|\alpha_0|^2) = -\alpha|x_{os}|^2 \frac{2}{3\pi}\left(\omega_{os}|\alpha_0|^2(b^2-a^2)+\omega_{os}^3|\alpha_0|^2 \ln\left|\frac{b^2-\omega_{os}^2}{\omega_{os}^2-a^2}\right|\right) \tag{4.1}$$

by considering both resonant and antiresonant contributions. Without going into the details of their theory, we would like to examine the claim that the first term of (4.1) generalizes the electromagnetic mass shift in the vacuum to include an observable dependence on $|\alpha_0|^2$, and to investigate the limitations of a two-level atomic model as opposed to a real atom.

Using the dipole sum rule on the first term of (4.1) in a generalization to many levels gives

$$\Delta E^{(1)}(|\alpha_0|^2) = \frac{2\alpha}{\pi m}\int \bar{n}_k k\, \mathrm{d}k = \Delta_1 \tag{4.2}$$

that is, the part quadratic in the field. So $\Delta(|\alpha_0|^2)$ should have a direct correspondence with our Δ_1 and Δ_2.

We had

$$\begin{aligned}\Delta_2 &= \frac{-4\alpha}{3\pi}\sum_n \Delta_{mn}^3|\boldsymbol{r}_{mn}|^2\int_0^\infty \frac{\bar{n}_k k\,\mathrm{d}k}{\Delta_{mn}^2-k^2}\\ &= \frac{-2\alpha}{3\pi}\sum_n \Delta_{mn}^3|\boldsymbol{r}_{mn}|^2 \ln\left(\frac{b^2-\Delta_{mn}^2}{\Delta_{mn}^2-a^2}\right)|\alpha_0|^2\end{aligned} \tag{4.3}$$

in agreement with Bullough and Caudrey. Now Δ_1, we have seen, shifts all levels equally. The fact that their equivalent term to Δ_1 does not do so is due to a separation of Δ_1 from Δ calculated using $\mathscr{H} = -e\boldsymbol{r}\cdot\boldsymbol{E}$, an effect peculiar to a two-level atom, since one then cannot use the dipole sum rule to remove the binding dependence. However, we have already pointed out the danger of referring to Δ_1 as a mass increase. The fact that Bullough and Caudrey extracted Δ_1 from an $e\boldsymbol{r}\cdot\boldsymbol{E}$ interaction is intriguing and we examine it from the point of view of (3.11) with such an interaction:

$$\begin{aligned}\Delta_{2S-2P} = \frac{-2}{3\pi}\int_0^\infty k^3\bar{n}_k\Bigg(&\sum_n \frac{|\langle ^2S|\mu|n\rangle|^2 2(E_n-E_{2S})}{k^2-(E_n-E_{2S})^2}\\ &-\sum_{n'}\frac{|\langle ^2P|\mu|n'\rangle|^2 2(E_{n'}-E_{2P})}{k^2-(E_{n'}-E_{2P})^2}\Bigg)\mathrm{d}k.\end{aligned} \tag{4.4}$$

Power (1966) has shown how we can evaluate (4.4) for the case of $\bar{n}_k = 1$ (ie the vacuum correction), and we follow his method. The Bethe result is derived from the above by making the assumption $k > (E_n - E_{2S,2P})$. We could thus expand (4.4) and show how the leading terms, by the dipole sum rule are just Δ_1 and thus cancel in Δ_{2S_2P} since they are independent of l. The remainder is the logarithmic term above, but needs rather cavalier treatment involving cut-offs (Welton 1948). Instead we may integrate (4.4) directly without any further assumptions about k other than the spectral profile. Again we get a leading term for both $\Delta(^2S)$ and $\Delta(^2P_{1/2})$ which is Δ_1 by the dipole sum rule and, since l independent, will cancel. The remainder is

$$\Delta_{2S\_2P}(\bar{n}_k) = \frac{-4|\alpha_0|^2}{3\pi}\left(\sum_n |\langle ^2S|\mu|n\rangle|^2 (E_n - E_{2S})^3 \frac{1}{2} \ln\left|\frac{b^2-(E_n-E_{2S})^2}{(E_n-E_{2S})^2-a^2}\right|\right.$$
$$\left. - \sum_{n'} |\langle ^2P|\mu|n'\rangle|^2 (E_{n'} - E_{2P})^3 \frac{1}{2} \ln\left|\frac{b^2-(E_{n'}-E_{2P})^2}{(E_{n'}-E_{2P})^2-a^2}\right|\right) \qquad (4.5)$$

which, when restricted to a two-level atom, is the second term of Bullough and Caudrey. There also exists the Δ(mixing) term, but we shall not calculate it explicitly since the restrictions laid down by the spectral profile are rather arbitrary and limiting. Suffice it to say that (4.5) is indeed the generalization of the Bethe formula, but of little experimental significance.

5. Effects of Lyman alpha photons

In any Lamb shift experiment, there are large numbers of Lyman alpha photons in the resonance region, arising out of the induced decay of $^2P_{1/2}$ atoms stimulated from the $^2S_{1/2}$ level by radio frequency transitions. Additional to these, there are often quite large numbers of L_α photons present from the excitation mechanism (for example in a level crossing experiment). These photons from the decay of one atom will interact with other atoms in the resonance region. In fact, quite apart from the type of shift considered here, such a mechanism is responsible for a collective phenomena known as radiation trapping which is dependent upon atomic densities and is known to shift fine structure levels (Stephen 1964) and this presents yet another environmental correction. Laying aside such complications, the effect of the L_α photons on the state $|m\rangle$ due to non-degenerate mixing is

$$\Delta = \frac{-4\alpha}{3\pi}\sum_n (E_n - E_m)|\boldsymbol{r}_{mn}|^2 \int_0^\infty \frac{k^3\, \mathrm{d}k\bar{n}_k}{k^2-(E_n-E_m)^2}$$
$$= -2\int_0^\infty \bar{n}_k \Pi(k^2) k^3\, \mathrm{d}k \qquad (5.1)$$

where $\Pi(k^2)$ is the dynamic polarizability. This is the light shift of Cohen-Tannoudji (1961). For $\Delta(^2S_{1/2})$ the nearest optically excited level is in the $n = 3$ term and the shift is not resonantly enhanced. For $\Delta(^2P_{1/2})$ due to $1^2S_{1/2}$ the shift is zero since the L_α is symmetric about the $2^2P_{1/2} - 1^2S_{1/2}$ decay frequency. Δ is thus likely to be too small to be of relevance. This is given support by the fact that there are likely to be of the order of 10^8 L_α photons in the resonance cavity, but these are distributed over a line-width of 10^8 Hz, so $\bar{n}_k$ is unlikely to be bigger than one unless the number of decaying atoms is greatly increased.

Similar arguments apply to Δ(mixing) which is

$$\Delta(\text{mixing}) = \frac{-4\alpha}{3\pi}(E_n - E_m) \int \frac{k^3 \, \mathrm{d}k \bar{n}_k}{(E_n - E_m)^2 - k^2} |\boldsymbol{r}_{mn}|^2. \tag{5.2}$$

Now $k \gg (E_n - E_m)$, so

$$\Delta(\text{mixing}) = \frac{4\alpha}{3\pi}(E_n - E_m)|\boldsymbol{r}_{mn}|^2 \int \bar{n}_k k \, \mathrm{d}k. \tag{5.3}$$

Now assuming again that $\bar{n}_k \lesssim 1$ then again we find a small effect. If, however, we are able to excite the hydrogenic atom using a laser so that $\bar{n}_k \gg 1$ this may well be a necessary correction. The largest part will come from the $^2P_{3/2}$ level, so substituting for $(E(^2P_{3/2}) - E(^2P_{1/2}))$ and $|\langle ^2S_{1/2}|\boldsymbol{r}|^2P_{3/2}\rangle|^2$ we have

$$\Delta(^2S_{1/2} \to {}^2P_{3/2}) = \frac{6}{8\pi} \frac{\alpha(Z\alpha)^2}{m} \int \bar{n}_k k \, \mathrm{d}k \tag{5.4}$$

and the final result will depend on the experimental photon spectral distribution.

Thus the interaction of the atom with the L_α excitation and decay photon is likely to be the most important correction, but it is still out of reach of present experimental techniques.

6. Comments

In any physically realizable situation, we have found that environmental corrections to the Lamb shift are always very small. Whilst the results are of no immediate experimental relevance, they are useful in indicating the care that is necessary in describing radiative level shifts and warn against generalizing just part of the interaction.

The reason for this is the combination and cancellation of terms in the very low energy region so that a bound electron at low energies interacts far less strongly than a free electron. We can modify the Lamb shift only by transitions to discrete levels at low energy whereas most of the Lamb shift in vacuum fields is due to continuum level transitions. At higher energies the electron scatters as if free and produces state independent level shifts which do not affect the vacuum Lamb shift. In the process of this work we have shown that the concept of a mass shift due to very low energy (less than the binding) fields is not a good one for a bound state, but a symptom of an incomplete description of the interaction.

Acknowledgments

After this calculation was completed I learnt that G Barton has arrived at similar results for the thermal corrections by a different method.

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